INMAS 2021, Modeling & Optimization Problems: Session 4.

1. Given a graph $G = (V, E)$, a spanning tree is a subgraph of G that has no cycles and that contains every vertex of G. A leaf of a tree is a node that is adjacent to only one edge of the tree. A maximum leaf spanning tree is a spanning tree with the maximum number of leaves.

   (a) Write an MILP formulation for the maximum leaf spanning tree problem. Hint: define variables

   $$x_e = \begin{cases} 1 & \text{if edge } e \text{ is included in the tree} \\ 0 & \text{otherwise.} \end{cases}$$

   Given a subset S of V, let $\bar{S}$ denote the complement of S, so that $\bar{S} = V \setminus S$. Let $E(S, \bar{S})$ denote the edges between S and $\bar{S}$. Then x describes a spanning tree if and only if x satisfies the following constraints:

   $$\sum_e x_e = n - 1 \tag{1}$$

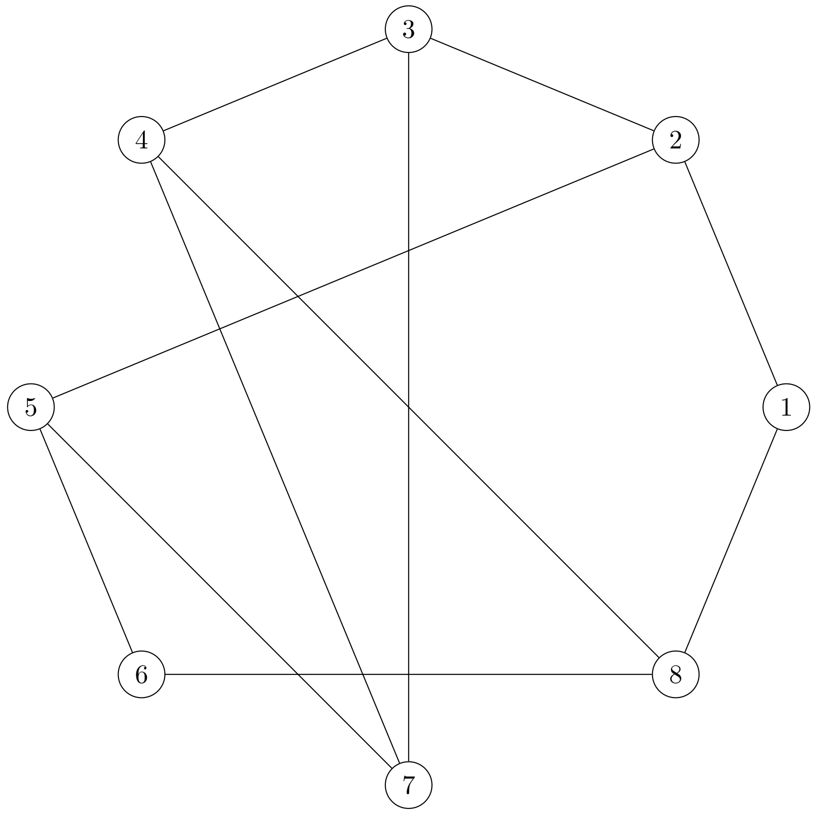
   $$\sum_{e \in E(S,\bar{S})} x_e \geq 1 \text{ for all } S \subsetneq V \tag{2}$$

   $$x_E \in \{0, 1\} \tag{3}$$

   The second set of constraints is similar to the subtour elimination constraints. Since a spanning tree connects all nodes of the graph, there must be at least one edge between each set S and its complement $\bar{S}$.

   (b) Suppose that you have an integral solution x. Suppose that x satisfies constraints (1) and (3), but x doesn't necessarily satisfy constraints (2). Describe a procedure that identifies a violated constraint if there are any.

   (c) Use Gurobi and constraint generation to implement a solution method for the maximum leaf spanning tree problem. Solve the following problem:



2. Consider the binary knapsack problem:

$$\max_x \sum_{i=1}^{n} c_i x_i$$

subject to:

$$\sum_{i=1}^{n} a_i x_i \leq B$$
$$x_i \in \{0, 1\}$$

(a) Describe a greedy heuristic for constructing a solution to this problem.

(b) Implement local search heuristic for this problem.

(c) Extend your local search heuristic to a tabu search heuristic.

(d) Extend your local search heuristic to a simulated annealing heuristic.

(e) Run each of your heuristics on some randomly generated problem instances. Solve these instances with Gurobi. How close to the optimal solution do your heuristics achieve?

3. Consider the k-center facility location problem. There is a set of customer locations L, as well as a set of possible facility locations F. We are allowed to open at most k facilities, and we want to minimize the longest distance that any customer has to travel to reach the nearest facility.

   (a) Describe an integer program formulation for this problem.

   (b) Describe a greedy heuristic for constructing a solution to this problem.

   (c) Implement local search heuristic for this problem.

   (d) Extend your local search heuristic to a tabu search heuristic.

   (e) Extend your local search heuristic to a simulated annealing heuristic.

   (f) Run each of your heuristics on some randomly generated problem instances. Solve these instances with Gurobi (Note: this problem is very challenging for IP solvers, so you may have to use very small instances). How close to the optimal solution do your heuristics achieve?